

# Relation between d-density wave of electron and staggered flux of spinon

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## Abstract

A  $d_{x^2-y^2}$ -density wave (ddw) order of electron in two-dimensional t-J model is analyzed in saddle point level using the U(1) slave boson formalism. We considered not only the staggered flux (s-flux) order of spinon but also the s-flux order of holon. This analysis provides the relation between the s-flux order of spinon and the ddw order of electron. We discovered a new phase in the phase diagram. In this phase, there is a s-flux order of spinon, but no ddw order of electron. Our results are that 1) a region of electron ddw exists, 2) there is no coexistence of ddw and  $d_{x^2-y^2}$ -wave pairing (singlet-RVB) in all region of phase diagram, and that 3) the ground state is a purely  $d_{x^2-y^2}$  wave superconducting state.

*Key words:* Superconductivity; Cuprate; Pseudo gap; d-density wave; t-J model

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Recently, Chakravarty et al. [1] proposed that the electron  $d_{x^2-y^2}$ -density wave (ddw) order exists in the pseudo gap region of high- $T_c$  superconductor. The electron ddw state [2] is the staggered flux (s-flux) state [3,4] of electron coordinate. In this state,  $d_{x^2-y^2}$  wave (d-wave) gap exists, time-reversal-symmetry is broken and the ‘real’ staggered current of the electron exists. The order parameter of the ddw is  $y_e = -i \sum_{\mathbf{k}\sigma} (\cos k_x - \cos k_y) \langle c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}+\mathbf{Q}\sigma} \rangle$ ,  $\mathbf{Q} = (\pi, \pi)$ . However they didn’t discuss this scenario microscopically. There remains a question, ‘can the electron ddw phase exist in highly correlated system?’.

The 2-dimensional t-J model is a promising model which includes highly correlated effects. Many phases are proposed in this model [3,4,5,6,7,8,9]. Zhang [8] analyzed the competition between s-flux order and  $d_{x^2-y^2}$ -wave pairing order at zero temperature by Gutzwiller approximation. The s-flux state is unstable against infinitesimal d-wave pairing at finite doping. Ubbens and Lee [9] analyzed this model at finite temperature. The s-flux phase of the spinon exists on the region where doping and temperature are both finite.

The SU(2) s-flux state, in which the Fermi-surface of spinon are always points, is also analyzed in SU(2) slave boson model [10,11]. However, there remains a question, ‘how do electrons behave in finite temperature s-flux phase?’. The current of the spinon and the electron are not equivalent in finite temperature s-flux phase because there is no Bose condensation of holon. The region of finite temperature s-flux phase exists above the temperature of holon condensation.

In this paper, we revealed the relation between the ddw of electron and the s-flux of spinon. We analyzed it in the 2-dimensional t-J model based on the U(1) slave boson formalism. We introduce order parameters  $\bar{\chi}_{ij} = \langle \sum_{\sigma} f_{i\sigma}^\dagger f_{j\sigma} \rangle$ ,  $\bar{\eta}_{ij} = \langle f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow} \rangle$ ,  $\bar{B}_{ij} = \langle b_i^\dagger b_j \rangle$  to decouple the Hamiltonian.

We considered not only the staggered flux order of the spinon but also the staggered flux order of the holon,  $\bar{\chi}_{i+\hat{x},i} = x_s + i(-1)^i y_s$ ,  $\bar{\chi}_{i+\hat{y},i} = x_s - i(-1)^i y_s$ ,  $\bar{B}_{i+\hat{x},i} = x_h + i(-1)^i y_h$ ,  $\bar{B}_{i+\hat{y},i} = x_h - i(-1)^i y_h$ . Here,  $\hat{x}$  and  $\hat{y}$  are unit vectors in the  $x$  and  $y$  direction,  $x_s = \chi \cos(\phi_s/4)$ ,  $y_s = \chi \sin(\phi_s/4)$ ,  $x_h = B \cos(\phi_h/4)$ ,  $y_h = B \sin(\phi_h/4)$ . The order parameters  $y_s$  and  $y_h$  correspond to the ddw order parameter of spinon and holon, respectively. For the pairing symmetry, we considered  $d_{x^2-y^2}$ , namely

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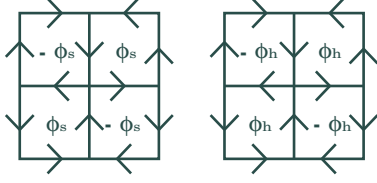


Fig. 1. Staggered flux of spinon  $\phi_s$  and holon  $\phi_h$ .

$$\bar{\eta}_{i+\hat{x},i} = -\bar{\eta}_{i+\hat{y},i} = \eta.$$

This formalism is an extension of the study by Ubbens and Lee [9]. The  $\sum_{\sigma} f_{j\sigma}^{\dagger} f_{i\sigma}$  term couples not only to  $(3J/8)\chi_{ij}$  but also to  $t\bar{B}_{ij}$ , i.e. the spinon feels the spinon s-flux( $\phi_s$ ) and the holon s-flux( $\phi_h$ ). The expectation values of the holon,  $B$  and  $\phi_h$ , have finite value for the solution of self-consistency equations. Two advantages exist in our formalism; 1) this is a new saddle point solution whose free energy is lower than the previous one, 2) this solution provides the relation between the ddw of the electron and the s-flux of the spinon. In this formalism, the hopping order parameter of the electron is a product of the hopping order parameters of spinon and holon.

$$\langle \sum_{\sigma} c_{i\sigma}^{\dagger} c_{j\sigma} \rangle = \langle \sum_{\sigma} f_{i\sigma}^{\dagger} f_{j\sigma} \rangle \langle b_j^{\dagger} b_i \rangle = \bar{\chi}_{ij} \bar{B}_{ij}^* \quad (1)$$

The electron s-flux order parameter  $\phi_e$  and the electron ddw order parameter  $y_e$  are given by  $\phi_e = \phi_s - \phi_h$ , and  $y_e = \chi B \sin(\phi_e/4)$ .

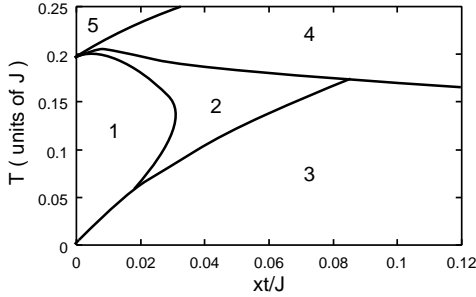


Fig. 2. MF phase diagram for  $t/J=1$ . The electron ddw order exists only in region 2. Here,  $x$  is hole concentration. The phase diagram for  $t/J=2$  is quantitatively similar to the phase diagram for  $t/J=1$ . With the boson order parameter  $B_{ij}$ , the  $\pi$ -flux phase and s-flux phase of spinon and holon extends to the higher-doped region compared to the previous work [9], where  $B_{ij}$  were not considered.

We solved the self-consistency equations numerically, and obtained the phase diagram (Fig. 2). At half-filling, the holon order parameter  $\bar{B}_{ij}$  is zero and the degeneracy of spinon between the s-flux state and the d-wave pairing state exists due to the local SU(2) symmetry;  $\chi \neq 0, \bar{B}_{ij} = 0, y_s^2 + \eta^2 = \text{const} \neq 0$ . In region 1, spinon s-flux exists but electron ddw doesn't exist. The spinon and holon state are  $\pi$ -flux

order state respectively. In electron picture, the s-flux is canceled completely;  $\chi \neq 0, B \neq 0, \phi_s = \phi_h = \pi, \eta = 0$ . The electron ddw order parameter,  $y_e = \chi B \sin((\pi - \pi)/4) = 0$ . The electron ddw order exists only in region 2. The staggered current of electron can be observed experimentally. The spinon s-flux  $\phi_s$  and holon s-flux  $\phi_h$  are not equal to  $\pi$  or 0, and  $\phi_s \neq \phi_h$ ;  $\chi \neq 0, B \neq 0, \phi_s \neq 0, \phi_h \neq 0, \eta = 0$ , and  $y_e = \chi B \sin((\phi_s - \phi_h)/4) \neq 0$ . In region 3,  $d_{x^2-y^2}$ -wave pairing exists;  $\chi \neq 0$  and  $B \neq 0, \phi_s = \phi_h = 0, \eta \neq 0$ , and  $y_e = 0$ . In region 4, there exists only uniform hopping order;  $\chi \neq 0, B \neq 0, \phi_s = \phi_h = \eta = 0$ , and  $y_e = 0$ . In region 5, all order parameters are zero. Spinon and holon cannot hop;  $\chi = B = \phi_s = \phi_h = \eta = 0$ , and  $y_e = 0$ .

The transition between region 1 and region 2 is a 2nd order transition in our theory. (If one only focuses on spinon degree of freedom, this doesn't look like phase transition [9,10,11].) There exists an order parameter which characterizes this transition. It is the electron ddw order parameter.

In conclusion we have found a possibility for the electron ddw phase in the t-J model at finite temperature, which does not coexist with the singlet-RVB state.

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